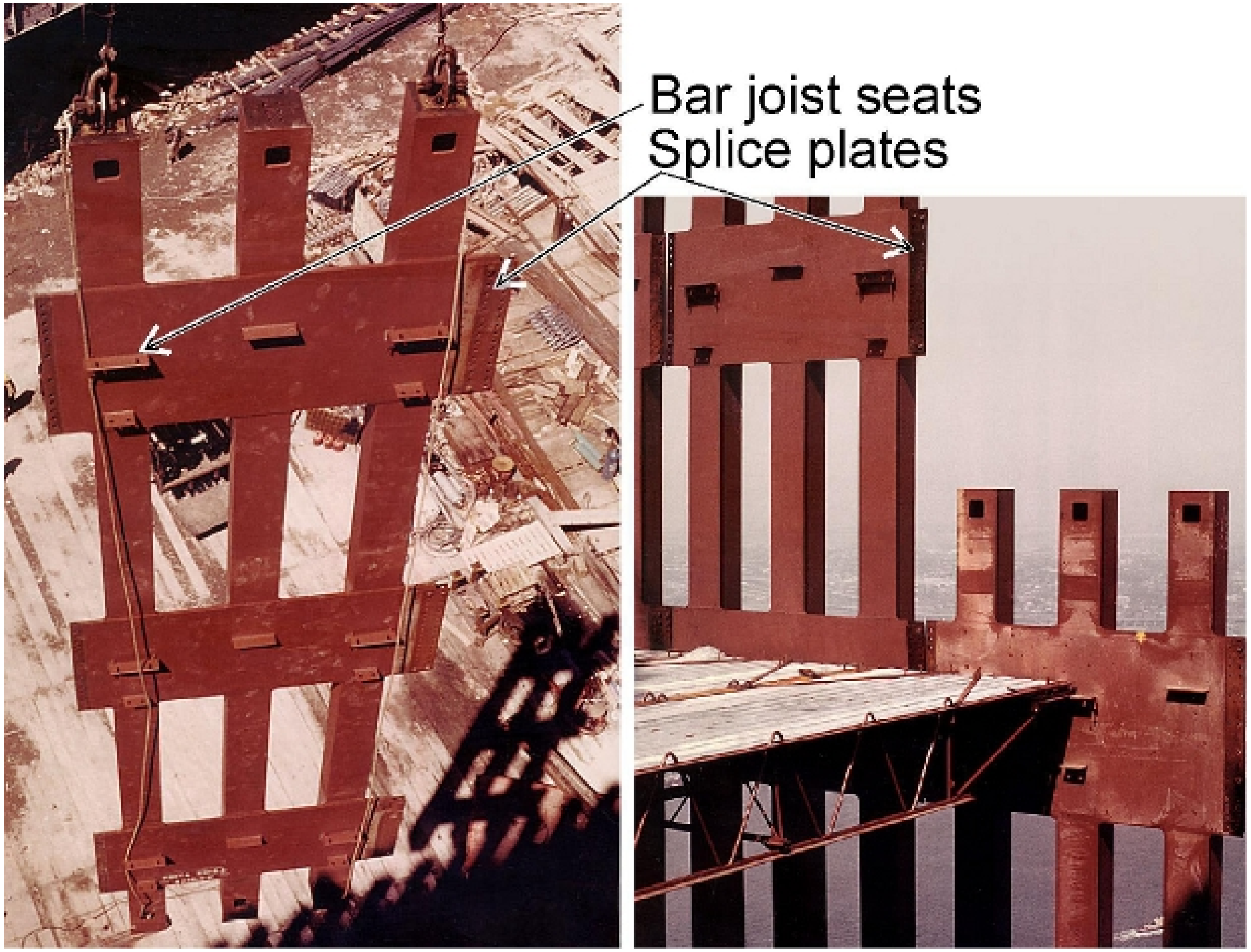
MOMENT OF INERTIA OF BUILT-UP SECTIONS



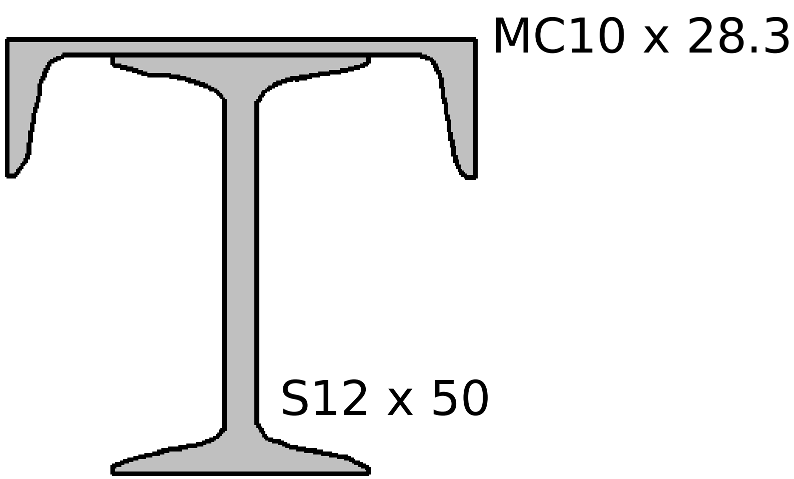
Construction of the World Trade Center
Perimeter Column Panels
Three Full Columns Connected by Three Spandrels

Moment of Inertia for Built-up Sections

- Frequently, standard structural sections are welded together to form a built-up section. The moments of inertia of each section are easily found in a handbook or from the vendor. However, as we saw in the section on moments of inertia for composite sections, we cannot algebraically add moments of inertia.
- Since 'built-up' section is simply another term for composite section, then finding moment of inertia for a built-up section is no different.
- Let's look at some examples of finding the moment of inertia of built-up sections.

Moment of Inertia for Built-up Sections

- Example 8:
  Consider a built-up section comprised of an S12 x 50 standard section capped with an MC10 x 28.3 miscellaneous channel. Determine the centroidal moment of inertia.

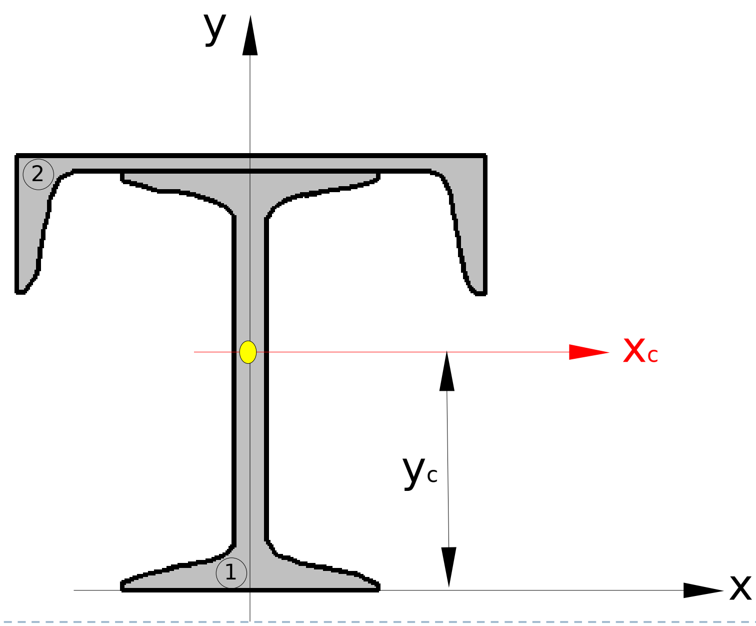


Moment of Inertia for Built-up Sections

- Finding any centroidal moment of inertia requires one knows the location of the centroid. Finding the centroid of a built-up section is no different from finding the centroid of a composite geometric section.
- First, impose an x-y coordinate axis at a location of your choice. In doing so, one should note it is appropriate to recognize if symmetry exists and to use it to your advantage. In the present example, the section is symmetrical about the y-axis but not about the x-axis. This places the x-coordinate of the centroid at the center of the section; this is a very good place to locate the y-axis of our imposed coordinate system.

Centroid of a Built-up Section

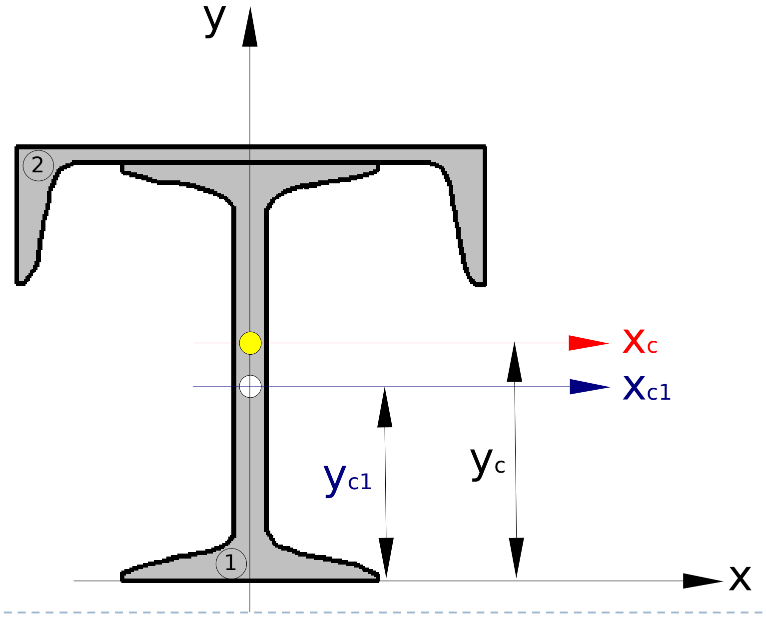
- Although we can place the x-axis anywhere we wish, locating it at the base of the structure is generally most convenient. The axis x_c is the centroidal x-axis, the location of which we wish to find.
- Now go to the appendix of the online text and determine the geometry and moments of inertia of each section.



(1) MC-Section	(2) S-Section
Web thickness: 0.477”	Depth: 12.00”
$\bar{X}_c$: 0.933”	Area: 14.7 in^2
I_y : 8.21 in^4	I_x: 305 in^4
Area : 8.32 in^2	

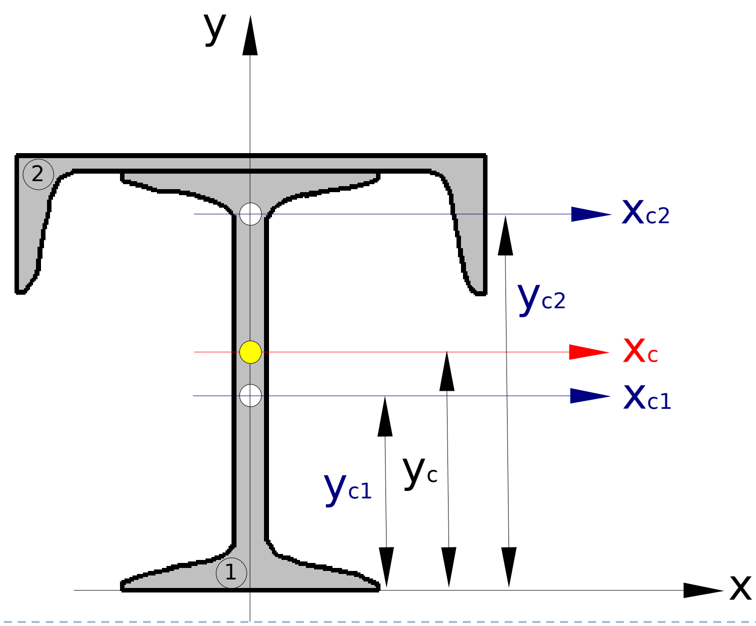
Centroid of a Built-up Section

- The location of the centroid for an 'I-beam'-type section is not explicitly stated in the tables. It is assumed you know it is equal to half the depth of the section due to symmetry.
- In this case, the centroid is located a distance y_{c1} from our arbitrary x-axis. That distance is 6.0".



Centroid of a Built-up Section

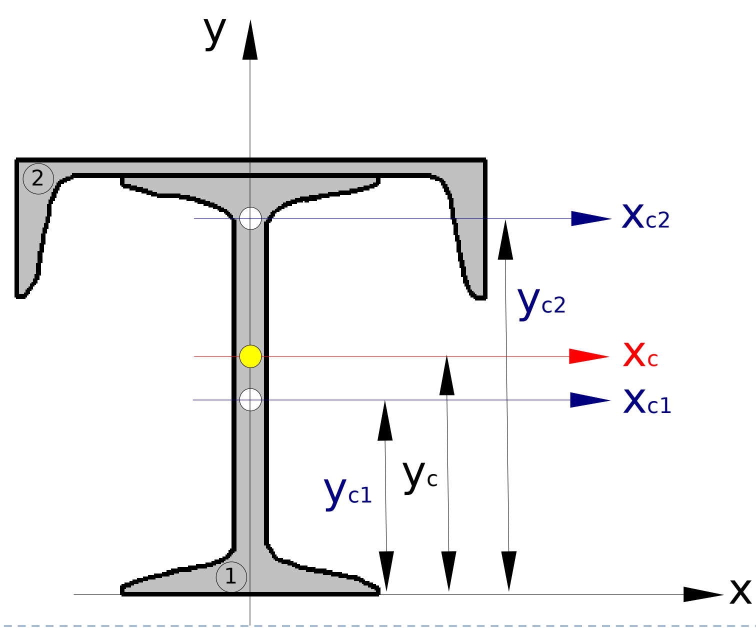
- The distance d_{y2} is a bit more tricky. The centroid of a channel section is measured from the back of the web and given as distance $\overline{X}_c$ in the steel tables.
- This distance must be measured from the same x-axis as the S-section. Thus, y_{c2} equals depth of the S-section plus the width of the channel web minus distance $\overline{X}_c$.



- This means:
  y_{c2} = 12” + 0.477” - 0.933”
  y_{c2} = 11.544”

Centroid of a Built-up Section

- Areas are also obtained from the steel tables.
- Once the areas and locations of centroids are obtained, we can calculate the location of the centroid.



Section	Area	y	yA
S	14.7	6.0	88.2
MC	8.32	11.544	96.05
Total	**23.02**	----	**184.2**

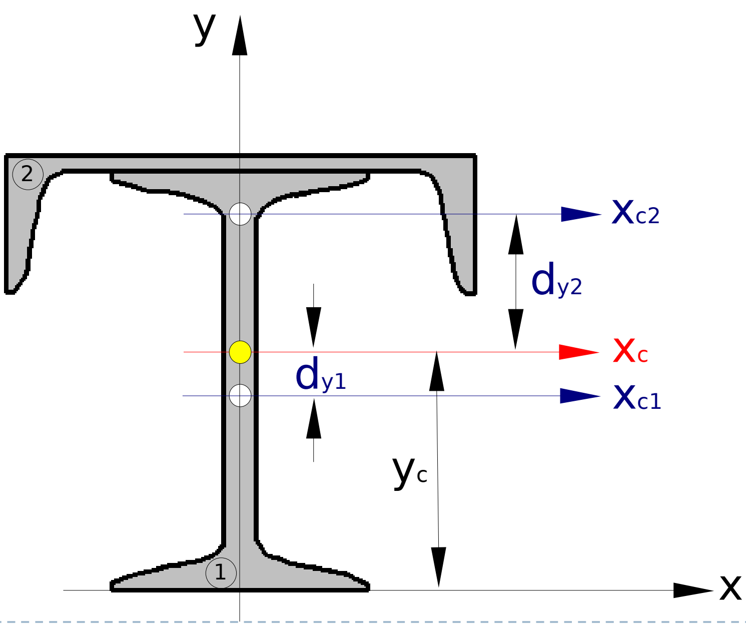
$$y_c = \frac{184.2\ \text{in}^3}{23.02\ \text{in}^2} = 8.0$$

Moment of Inertia of a Built-up Section

- Once the centroid is located, the moment of inertia is found by applying the parallel axis theorem.
- The distances between the centroidal axis of the composite section and the individual sections are easily calculated as:

$$d_{y1} = 8 - 6 = 2 \text{ in}$$
$$d_{y2} = 11.544 - 8 = 3.544 \text{ in}$$

The remainder of the calculations are expressed in the table below



Section	Area	Ix	$d_{y1,2}$	$d^2_{y1,2}$ (A)	Ix + $d^2_{y1,2}$ (A)
S	14.7	305	8 - 6	58.80	364
MC	8.32	8.21	11.54 - 8	104.26	112
Total	**23.02**	----	----	----	**476**

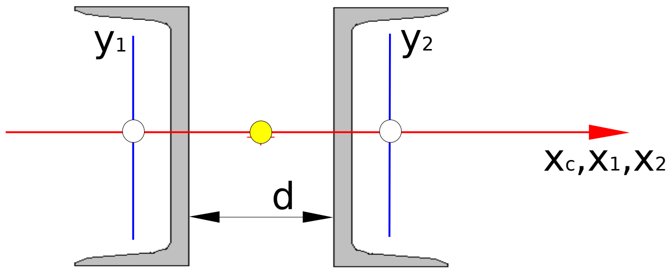
Moment of Inertia of a Built-up Section

- Example 9:
  A column will tend to buckle about the axis with the least moment of inertia. For that reason, it is preferred the moment of inertia about the x- and y-axis of a column section to be roughly equal.
- Consider a built-up column comprised of two MC12x35 channels. Determine the distance 'd' between the sections such that the centroidal moment of inertia about the x- and y-axis are equal.
- From the appendix, we can lookup the following data.

Area: $10.3\ in^2$	I_x: $216\ in^4$
x_c: 1.05 in	I_y: $12.7\ in^4$
t_w: 0.467 in	

Moment of Inertia of a Built-up Section

- Since both sections have the same x-axis, we know the composite moment of inertia about the x-axis is the sum of the moments of inertia of each individual section about its own centroidal x-axis. This gives us our desired I_y.
- However, we will need to apply the parallel axis theorem to find distance 'd' that will result in I_y.



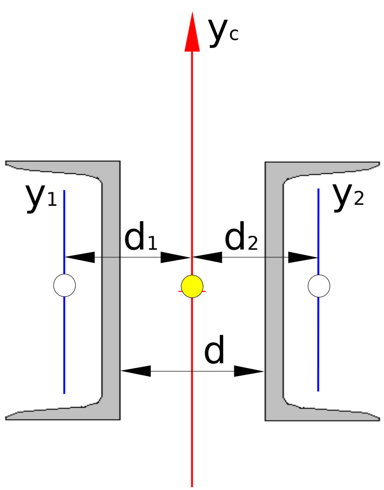
$$I_x = I_{x1} + I_{x2}$$
$$= 216 + 216 = 432 \text{ in}^4$$

Now we know that:

$$I_y = I_x = 432 \text{ in}^4$$

Moment of Inertia of a Built-up Section

- Since the section is symmetrical about the y-axis, we know that $d_1 = d_2 = ½\, d + t_w + x_c = ½\, d + 1.517$
- The moment of inertia of each section as it references the common centroidal y-axis must be half of the desired I_y. Applying the parallel axis theorem, we can write:



$$I_{each(y)} = \frac{I_y}{2} = 216 \text{ in}^4 = I_{y1} + d_1^2 \cdot A$$

$$= 12.7 + (0.5\text{d} + 1.517)^2 \cdot 10.3 \text{ in}^2$$

$$0 = 2.575 \cdot d^2 + 15.625 \cdot d - 179.60$$

Applying the quadratic equation:

$$d = \frac{-15.625 \pm \sqrt{15.625^2 - 4 \cdot 2.575 \cdot (-179.60)}}{2 \cdot 2.575}$$

$$d = 5.85 \text{ in}$$